QUIZ #2 – Solutions
Each problem is worth 5 points

15 points total

1.

When we write $\dfrac{dy}{dx} = \dfrac{y/x+1}{y/x-1}$, the differential equation is clearly homogeneous. We therefore set $v = y/x$, or, $y = vx$, in which case $dy/dx = v + x dv/dx$, and $v + x\dfrac{dv}{dx} = \dfrac{v+1}{v-1}$. This can be separated in the form $\dfrac{v-1}{-v^2+2v+1}\,dv = \dfrac{1}{x}\,dx$. A one-parameter family of solutions of this equation is defined implicitly by $-(1/2)\ln|-v^2+2v+1| = \ln|x| + C$. When this equation is exponentiated, $-v^2+2v+1 = D/x^2$, and substitution of $v = y/x$ gives $x^2 + 2xy - y^2 = D$.

2.

If we set $z = 1/y$, then $dz/dx = (-1/y^2)dy/dx$, and $-y^2\dfrac{dz}{dx} + y = y^2e^x$. Division by $-y^2$ gives $\dfrac{dz}{dx} - \dfrac{1}{y} = -e^x \implies \dfrac{dz}{dx} - z = -e^x$. An integrating factor for this equation is $e^{\int -dx} = e^{-x}$. When we multiply the differential equation by this factor, $\dfrac{d}{dx}(ze^{-x}) = -1$. Integration now yields $ze^{-x} = -x + C \Longrightarrow z = (C-x)e^x$. Thus, $\dfrac{1}{y} = (C-x)e^x \Longrightarrow y = \dfrac{e^{-x}}{C-x}$.

3.

We could use substitution 16.28 but it simpler to write

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = k \quad \Longrightarrow \quad r\frac{dT}{dr} = kr + C \quad \Longrightarrow \quad \frac{dT}{dr} = k + \frac{C}{r} \quad \Longrightarrow \quad T = kr + C\ln r + D.$$

For $T(a) = T_a$ and $T(b) = T_b$, C and D must satisfy $T_a = ka + C\ln a + D$ and $T_b = kb + C\ln b + D$. These can be solved for $C = [T_b - T_a - k(b-a)]/\ln(b/a)$ and $D = [T_a\ln b - T_b\ln a + k(b\ln a - a\ln b)]/\ln(b/a)$.